# Differentiation Formulas Explained

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**Definition:** Derivative of a function $f(x)$ with respect to $x$ is $f'(x)$ given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \tag{2}$$

**Theorem 1.** If $f(x)$ is differentiable at $a$ then $f(x)$ is continuous at $a$

*Proof.* Since $f(x)$ is differentiable at $x = a$, the limit defined by equation 2 exists.

$$f(x) - f(a) = \frac{f(x) - f(a)}{(x - a)}(x - a)$$

$$\lim_{x\to a}(f(x) - f(a)) = \lim_{x\to a}(\frac{f(x) - f(a)}{(x - a)}(x - a))$$

$$= \lim_{x\to a}(\frac{f(x) - f(a)}{(x - a)}) \lim_{x\to a}(x - a)$$

Since $\lim_{x\to a}(x - a) = 0$

$$\lim_{x\to a}(f(x) - f(a)) = f'(a) * 0 = 0 \tag{3}$$

$$\lim_{x\to a} f(x) = \lim_{x\to a}[f(x) + f(a) - f(a)]$$

$$= \lim_{x\to a}[f(a) + f(x) - f(a)]$$

$$= \lim_{x\to a} f(a) + \lim_{x\to a}[f(x) - f(a)]$$

Using equation 3

$$= \lim_{x\to a} f(a) + 0$$

$$= f(a)$$

$$\Rightarrow \lim_{x\to a} f(x) = f(a)$$

Since $\lim_{x\to a} f(x) = f(a)$, the function $f(x)$ is continous at $a$ $\square$

**Derivative of sum and difference of two functions.**

$$y = f(x) \pm g(x)$$

*Proof.*

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h\to 0} \frac{f(x+h) \pm g(x+h) - f(x) \mp g(x))}{h} \\
&= \lim_{h\to 0} \frac{f(x+h) - f(x) \pm g(x+h) \mp g(x))}{h} \\
&= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \pm \lim_{h\to 0} \frac{g(x+h) - g(x))}{h} \\
&= f'(x) \pm g'(x) \\
\Rightarrow \frac{dy}{dx} &= f'(x) \pm g'(x)
\end{aligned}$$

□

**Derivative of constant times a function.**

$$y = cf(x)$$

*Proof.*

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h\to 0} \frac{c * f(x+h) - c * f(x)}{h} \\
&= c * \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= c * f'(x) \\
\Rightarrow \frac{dy}{dx} &= c * f'(x)
\end{aligned}$$

□

**Derivative of a constant function.**

$$y = f(x) = c$$

*Proof.*

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{c - c}{h} \\
&= \lim_{h\to 0} 0 \\
\Rightarrow f'(x) &= 0
\end{aligned}$$

□

**Power Rule** We are going to derive the power rule using Binomial Theorem $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} * b^k$

$$y = x^n$$

*Proof.*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

Expand $(x+h)^n$using Binomial Theorem

$$= \lim_{h\to 0} \sum_{k=0}^{n} \binom{n}{k} \frac{a^{n-k} * b^k}{-} x^n h$$
$$= \lim_{h\to 0} \frac{x^n + n * x^{n-1} * h + \frac{n*(n-1)}{2!} x^{n-2} * h^2 + \ldots + n * x * h^{n-1} + h^n - x^n}{h}$$
$$= \lim_{h\to 0} \frac{x^n - x^n + h * n * x^{n-1} + \frac{n*(n-1)}{2!} x^{n-2} * h + \ldots + n * x * h^{n-2} + h^{n-1}}{h}$$
$$= \lim_{h\to 0} n * x^{n-1} + \frac{n*(n-1)}{2!} x^{n-2} * h + \ldots + n * x * h^{n-2} + h^{n-1}$$
$$= \lim_{h\to 0} n * x^{n-1} + \lim_{h\to 0} \frac{n*(n-1)}{2!} x^{n-2} * h + \ldots + n * x * h^{n-2} + h^{n-1}$$
$$= n * x^{n-1} + 0$$
$$\Rightarrow f'(x) = n * x^{n-1}$$

□

**Product Rule**

$$y = f(x)g(x)$$

*Proof.*

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$
$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$
$$= \lim_{h\to 0} \frac{f(x+h)(g(x+h) - g(x)) + g(x)(f(x+h) - f(x))}{h}$$
$$= \lim_{h\to 0} \frac{f(x+h)(g(x+h) - g(x))}{h} + \lim_{h\to 0} \frac{g(x)(f(x+h) - f(x))}{h}$$
$$= \lim_{h\to 0} f(x+h) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h\to 0} g(x) \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= f(x)g'(x) + g(x)f'(x)$$
$$\Rightarrow \frac{dy}{dx} = f(x)g'(x) + g(x)f'(x)$$

□

**Quotient Rule**

$$y = \frac{f(x)}{g(x)}$$

*Proof.*

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h\to 0}\left(\left(\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}\right)/h\right)\\
&= \lim_{h\to 0}\left(\left(\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}\right)/h\right)\\
&= \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{g(x+h)g(x)h}\\
&= \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x)}{g(x+h)g(x)h} + \lim_{h\to 0}\frac{f(x)g(x) - f(x)g(x+h)}{g(x+h)g(x)h}\\
&= \lim_{h\to 0}\frac{1}{g(x+h)g(x)}\lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x)}{h}\\
&\qquad - \lim_{h\to 0}\frac{1}{g(x+h)g(x)}\lim_{h\to 0}\frac{f(x)g(x+h) - f(x)g(x)}{h}\\
&= \frac{1}{g(x)g(x)}\lim_{h\to 0}\frac{g(x)(f(x+h) - f(x))}{h}\\
&\qquad - \frac{1}{g(x)g(x)}\lim_{h\to 0}\frac{f(x)(g(x+h) - g(x))}{h}\\
&= \frac{1}{g(x)^2}\lim_{h\to 0}g(x)\lim_{h\to 0}\frac{g(x)(f(x+h) - f(x)}{h}\\
&\qquad - \frac{1}{g(x)^2}\lim_{h\to 0}f(x)\lim_{h\to 0}\frac{g(x+h) - g(x)}{h}\\
&= \frac{1}{g(x)^2}g(x)f'(x) - \frac{1}{g(x)^2}f(x)g'(x)\\
&= \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}\\
\Rightarrow \frac{dy}{dx} &= \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}
\end{aligned}$$

□

**Chain Rule**

$$y = f{\circ}g(x)$$

*Proof.*

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h} \tag{4}$$

In the above equation we need to replace f(g(x+h)) in terms of f(x) and g(x).
So letś consider the derivation of inner function g(x) and outer function f(x)

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$\Rightarrow \frac{g(x+h) - g(x)}{h} - g'(x) \to 0 \text{ as } h \to 0$$

$$v(x) = \begin{cases} \frac{g(x+h)-g(x)}{h} - g'(x), & x \neq 0 \\ 0, & x = 0 \end{cases} \tag{5}$$

$$Similarly, w(x) = \begin{cases} \frac{f(x+k)-f(x)}{k} - f'(x), & x \neq 0 \\ 0, & x = 0 \end{cases} \tag{6}$$

From (5) and (6)

$$g(x+h) = g(x) + (g'(x) + v(x))h \tag{7}$$

$$f(x+k) = f(x) + (f'(x) + w(x))k \tag{8}$$

Using (7) and (8), we can write f(g(x+h) as

$$f(g(x+h)) = f(g(x)) + (f'(g(x)) + w(x))(g'(x) + v(x))h \tag{9}$$

where x=g(x) and k=(g'(x)+v(x))h in f(x+k)

Using (9) in (4)

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(g(x)) + (f'(g(x)) + w(x))(g'(x) + v(x))h - f(g(x))}{h}$$

$$= \lim_{h\to 0} \frac{(f'(g(x)) + w(x))(g'(x) + v(x))h}{h}$$

$$= \lim_{h\to 0} (f'(g(x)) + w(x)) \lim_{h\to 0} (g'(x) + v(x))$$

$v(x) \to 0$ and $w(x) \to 0$ as $h \to 0$

$$= (f'(g(x)) + 0)(g'(x) + 0)$$

$$\Rightarrow \frac{dy}{dx} = f'(g(x)) + g'(x)$$

□

**Derivative of Exponential function.** The derivative of $e^x$ can be found in different ways. I like using the expansion series of $e^x$ to find its derivative,

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

*Proof.*

$$\begin{aligned}
y = e^x &= \sum_{k=0}^{\infty} \frac{x^k}{k!} \\
\Rightarrow \frac{dy}{dx} &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x * e^h - e^x}{h} \\
&= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} \\
&= e^x \lim_{h\to 0} \frac{e^h - 1}{h}
\end{aligned}$$

Expanding $e^h$

$$\begin{aligned}
&= e^x \lim_{h\to 0} \frac{(1 + h + h^2/2! + h^3/3! + h^4/4! + ...) - 1}{h} \\
&= e^x \lim_{h\to 0} \frac{h(1 + h/2! + h^2/3! + h^3/4! + ...)}{h} \\
&= e^x \lim_{h\to 0} (1 + h/2! + h^2/3! + h^3/4! + ...) - 1 \\
&= e^x * 1 \\
\Rightarrow \frac{dy}{dx} &= e^x
\end{aligned}$$

□

*Note.* It can be seen from the above proof that

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

where e is a unique number for which this limit equals 1. □

**Inverse Rule** This rule will be very useful to evaluate the derivative of a function when its inverse function's derivative is known. Consider a function $f(x)$ whose inverse function is $f^{-1}(x)$ and we know that $f^{-1}(f(x)) = x$ or $f(f^{-1}(x)) = x$

*Proof.*

$$\begin{aligned}
y &= f^{-1}(x) \\
\Rightarrow f(y) &= x \\
f'(y)y' &= 1 \text{ Using chain rule} \\
y' &= \frac{1}{f'(y)} \\
&= \frac{1}{f'(f^{-1}(x))} \\
\Rightarrow \frac{d}{dx}(f^{-1}(x)) &= \frac{1}{f'(f^{-1}(x))} \qquad (10)
\end{aligned}$$

□

**Derivative of Logarithmic function.**

$$y = log(x)$$

$log(x)$ is the inverse function of $e^x$. Using $e^x$ in (10) we find the derivative of $log(x)$

*Proof.*

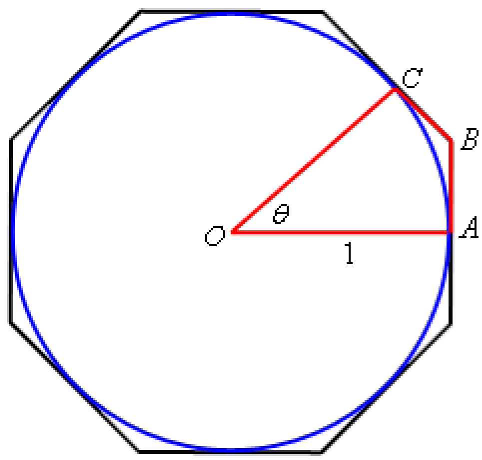
$$\begin{aligned}
\frac{d}{dx}(log(x)) &= \frac{1}{e^{log(x)}} \\
\text{We know that } e^{log(x)} &= x \\
&= \frac{1}{x} \\
\Rightarrow \frac{d}{dx}(log(x)) &= \frac{1}{x}
\end{aligned}$$

□

**Trignometric Basic** Before finding the derivative of trigonometric functions, we shall find the value of the following functions, $f(x) = \lim_{\theta \to 0} \frac{sin(\theta)}{\theta}$ and $f(x) = \lim_{\theta \to 0} \frac{cos(\theta)-1}{\theta}$, which will be useful in evaluating the drivative of trigonometric functions.

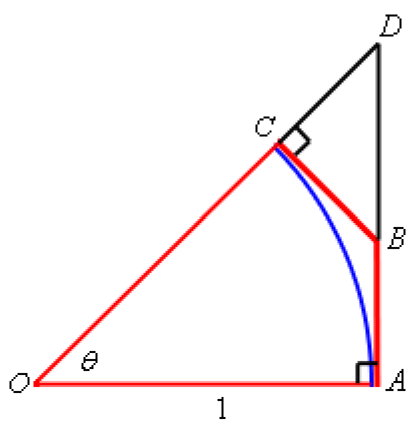
$$f(x) = \lim_{\theta \to 0} \frac{sin(\theta)}{\theta} = 1$$

*Proof.* To evaluate this we use Squuze Theorem. What we try to do is squuze the arc AC between ABC and straight line AC




$$arcAC < |AB| + |BC|$$

Extend AB to meet OC at D

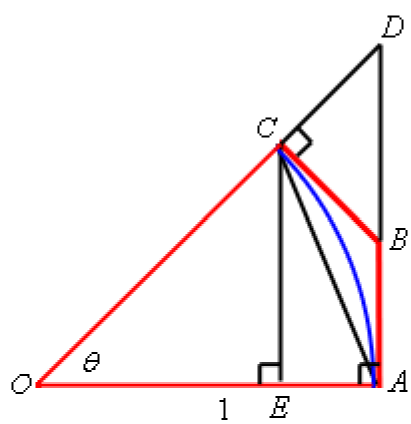


$$\Rightarrow arcAC < |AB| + |BD| = |AD|$$
$$arcAC < |AD|$$
$$Also, tan(\theta) = \frac{AD}{OA}$$
$$AD = tan(\theta) * 1$$
$$\Rightarrow arcAC < tan(\theta) \tag{11}$$

Length of arc AC is given by

$$arcAC = |OA|\theta$$
$$arcAC = \theta \tag{12}$$
$$Using (11) and (12),$$
$$\theta = arcAC < tan(\theta)$$
$$\theta < tan(\theta)$$
$$\theta < \frac{sin(\theta)}{cos(\theta)}$$
$$cos(\theta) < \frac{sin(\theta)}{\theta} \tag{13}$$

Next, in the above diagram add lines AC and CE perpendicular to OA



$$|CE| < |AC| < arcAC \tag{14}$$

$$|CE| = sin(\theta)|OC|$$

$$|CE| = sin(\theta) \tag{15}$$

Using (15) in (14)

$$sin(\theta) < arcAC$$

$$sin(\theta) < \theta$$

$$\frac{sin(\theta)}{\theta} < 1 \tag{16}$$

From (13) in (16)

$$cos(\theta) < \frac{sin(\theta)}{\theta} < 1 \text{ ,where } 0 \leq \theta \leq \pi/2 \tag{17}$$

Also , $\lim_{\theta \to 0} cos(\theta) = 1$ and $\lim_{\theta \to 0} 1 = 1$

So, by squeeze theorem $\lim_{\theta \to 0^+} \frac{sin(\theta)}{\theta} = 1$

Since sin(x) is odd function , $\frac{sin(-\theta)}{-\theta} = \frac{-sin(\theta)}{-\theta}$

$$= \frac{sin(\theta)}{\theta}$$

So, we can say $\lim_{\theta \to 0^-} \frac{sin(\theta)}{\theta} = 1$

$$\Rightarrow \lim_{\theta \to 0} \frac{sin(\theta)}{\theta} = 1$$

□

$$f(x) = \lim_{\theta \to 0} \frac{cos(\theta) - 1}{\theta} = 0$$

*Proof.*

$$\begin{aligned}
\lim_{\theta \to 0} \frac{cos(\theta) - 1}{\theta} &= \lim_{\theta \to 0} \frac{(cos(\theta) - 1)(cos(\theta) + 1)}{\theta(cos(\theta) + 1)} \\
&= \lim_{\theta \to 0} \frac{cos^2(\theta) - 1}{\theta(cos(\theta) + 1)} \\
&= \lim_{\theta \to 0} \frac{-sin^2(\theta)}{\theta(cos(\theta) + 1)} \\
&= \lim_{\theta \to 0} \frac{sin(\theta)}{\theta} \frac{-sin(\theta)}{(cos(\theta) + 1)} \\
&= \lim_{\theta \to 0} \frac{sin(\theta)}{\theta} \lim_{\theta \to 0} \frac{-sin(\theta)}{(cos(\theta) + 1)} \\
&= 1 * 0 \\
\Rightarrow \lim_{\theta \to 0} \frac{cos(\theta) - 1}{\theta} &= 0
\end{aligned}$$

□

**Derivative of sin(x).**

$$y = sin(x)$$

*Proof.*

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h \to 0} \frac{sin(x + h) - sin(x)}{h} \\
&= \lim_{h \to 0} \frac{cos(x)sin(h) + sin(x)cos(h) - sin(x)}{h} \\
&= \lim_{h \to 0} \frac{cos(x)sin(h) + sin(x)(cos(h) - 1)}{h} \\
&= \lim_{h \to 0} \frac{cos(x)sin(h)}{h} + \lim_{h \to 0} \frac{sin(x)(cos(h) - 1)}{h} \\
&= \lim_{h \to 0} cos(x) \lim_{h \to 0} \frac{sin(h)}{h} + \lim_{h \to 0} sin(x) \lim_{h \to 0} \frac{cos(h) - 1}{h} \\
&= cos(x) * 1 + sin(x) * 0 \\
\Rightarrow \frac{dy}{dx} &= cos(x)
\end{aligned}$$

□

**Derivative of cos(x).**

$$y = cos(x)$$

*Proof.*

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h\to 0} \frac{cos(x+h) - cos(x)}{h} \\
&= \lim_{h\to 0} \frac{cos(x)cos(h) - sin(x)sin(h) - cos(x)}{h} \\
&= \lim_{h\to 0} \frac{cos(x)(cos(h)-1) - sin(x)sin(h)}{h} \\
&= \lim_{h\to 0} \frac{cos(x)(cos(h)-1)}{h} + \lim_{h\to 0} \frac{sin(x)sin(h)}{h} \\
&= \lim_{h\to 0} cos(x) \lim_{h\to 0} \frac{cos(h)-1}{h} - \lim_{h\to 0} sin(x) \lim_{h\to 0} \frac{sin(h)}{h} \\
&= cos(x) * 0 - sin(x) * 1 \\
\Rightarrow \frac{dy}{dx} &= -sin(x)
\end{aligned}$$

□

**Derivative of tan(x).**

$$y = tan(x)$$

*Proof.*

$$tan(x) = \frac{sin(x)}{cos(x)}$$

Using quotient rule

$$\begin{aligned}
\frac{dy}{dx} &= \frac{cos(x)cos(x) - sin(x)(-sin(x))}{cos^2(x)} \\
&= \frac{cos^2(x) + sin^2(x)}{cos^2(x)} \\
&= \frac{1}{cos^2(x)} \\
\Rightarrow \frac{dy}{dx} &= sec^2(x)
\end{aligned}$$

□

**Derivative of csc(x).**

$$y = csc(x)$$

*Proof.*

$$csc(x) = \frac{1}{sin(x)}$$

Using quotient rule

$$\frac{dy}{dx} = \frac{sin(x) * 0 - 1(cos(x))}{sin^2(x)}$$

$$= \frac{-cos(x)}{sin^2(x)}$$

$$= \frac{-1}{sin(x)} \frac{cos(x)}{sin(x)}$$

$$\Rightarrow \frac{dy}{dx} = -csc(x)cot(x)$$

□

**Derivative of sec(x).**

$$y = sec(x)$$

*Proof.*

$$sec(x) = \frac{1}{cos(x)}$$

Using quotient rule

$$\frac{dy}{dx} = \frac{cos(x) * 0 - 1(-sin(x))}{cos^2(x)}$$

$$= \frac{sin(x)}{cos^2(x)}$$

$$= \frac{sin(x)}{cos(x)} \frac{1}{cos(x)}$$

$$\Rightarrow \frac{dy}{dx} = tan(x)sec(x)$$

□

**Derivative of cot(x).**

$$y = cot(x)$$

*Proof.*

$$cot(x) = \frac{cos(x)}{sin(x)}$$

Using quotient rule

$$\begin{aligned}
\frac{dy}{dx} &= \frac{sin(x)(-sin(x)) - cos(x)(cos(x))}{sin^2(x)} \\
&= \frac{1 - (sin^2(x) + cos^2(x))}{sin^2(x)} \\
&= \frac{-1}{sin^2(x)} \\
\Rightarrow \frac{dy}{dx} &= -csc^2(x)
\end{aligned}$$

□

**Derivative of arcsin(x).**

$$y = sin^{-1}(x)$$

*Proof.*

Using chain rule

$$\begin{aligned}
sin(y) &= x \\
cos(y)\frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{1}{cos(y)} \\
&= \frac{1}{\sqrt{1 - sin^2(y)}} \\
\Rightarrow \frac{dy}{dx} &= \frac{1}{\sqrt{1 - x^2}}
\end{aligned}$$

□

**Derivative of arccos(x).**

$$y = cos^{-1}(x)$$

*Proof.*

$$\begin{aligned}
&\text{Using chain rule}\\
cos(y) &= x\\
-sin(y)\frac{dy}{dx} &= 1\\
\frac{dy}{dx} &= \frac{1}{-sin(y)}\\
&= \frac{-1}{\sqrt{1 - cos^2(y)}}\\
\Rightarrow \frac{dy}{dx} &= \frac{-1}{\sqrt{1 - x^2}}
\end{aligned}$$

□

**Derivative of arctan(x).**

$$y = tan^{-1}(x)$$

*Proof.*

$$\begin{aligned}
&\text{Using chain rule}\\
tan(y) &= x\\
sec^2(y)\frac{dy}{dx} &= 1\\
\frac{dy}{dx} &= \frac{1}{sec^2(y)} \qquad (18)\\
sec^2(y) &= \frac{1}{cos^2(y)}\\
sec^2(y) &= \frac{cos^2(y) + sin^2(y)}{cos^2(y)}\\
sec^2(y) &= \frac{cos^2(y)}{cos^2(y)} + \frac{sin^2(y)}{cos^2(y)}\\
sec^2(y) &= 1 + tan^2(y)\\
&\text{Using this in (18)}\\
\frac{dy}{dx} &= \frac{1}{1 + tan^2(y)}\\
\Rightarrow \frac{dy}{dx} &= \frac{1}{1 + x^2}
\end{aligned}$$

□

**Derivative of arccsc(x).**

$$y = csc^{-1}(x)$$

*Proof.*

$$\begin{aligned}
csc(y) &= x \\
\frac{1}{sin(y)} &= x \\
\frac{-cos(y)}{sin^2(y)}\frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{sin^2(y)}{-\sqrt{1 - sin^2(y)}} \\
&= -\frac{(1/x)^2}{\sqrt{1 - (1/x)^2}} \\
&= -\frac{1}{x^2\sqrt{\frac{x^2-1}{x^2}}} \\
\Rightarrow \frac{dy}{dx} &= -\frac{1}{\frac{x^2}{|x|}\sqrt{x^2 - 1}} = -\frac{1}{|x|\sqrt{x^2 - 1}}
\end{aligned}$$

□

**Derivative of arccsc(x).**

$$y = sec^{-1}(x)$$

*Proof.*

$$\begin{aligned}
sec(y) &= x \\
\frac{1}{cos(y)} &= x \\
\frac{sin(y)}{cos^2(y)}\frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{cos^2(y)}{\sqrt{1 - cos^2(y)}} \\
&= \frac{(1/x)^2}{\sqrt{1 - (1/x)^2}} \\
&= \frac{1}{x^2\sqrt{\frac{x^2-1}{x^2}}} \\
\Rightarrow \frac{dy}{dx} &= \frac{1}{\frac{x^2}{|x|}\sqrt{x^2 - 1}} = \frac{1}{|x|\sqrt{x^2 - 1}}
\end{aligned}$$

□

**Derivative of arccot(x).**

$$y = cot^{-1}(x)$$

*Proof.*

$$\begin{aligned}
cot(y) &= x \\
\frac{cos(y)}{sin(y)} &= x \\
\frac{-(sin^2(y) + cos^2(y))}{sin^2(y)}\frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{-sin^2(y)}{sin^2(y) + cos^2(y)} \\
&= -1/\frac{sin^2(y) + cos^2(y)}{sin^(y)} \\
&= -\frac{1}{1 + cot^2(y)} \\
\Rightarrow \frac{dy}{dx} &= -\frac{1}{1 + x^2}
\end{aligned}$$

□